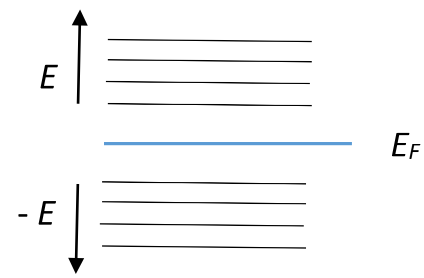
Fermi-Dirac distribution

The probability that the available energy state '*E*' will be occupied by an electron at absolute temperature *T* under conditions of thermal equilibrium is given by the Fermi-Dirac function. From quantum physics, the Fermi-Dirac function is

$$f(E) = \frac{1}{1 + exp\left[\frac{E - E_F}{\mathrm{k_B}\, T}\right]}$$

where $\mathrm{k_B}$ is the Boltzmann constant, *T* is the temperature in ^{0}**K** and $\boldsymbol{E_F}$ is the Fermi energy level in eV. $\mathrm{k_B}$= 1.38 X 10^{-23} J/K

Three cases are shown below to determine the probability of finding electron:



Case 1: the probability of finding electron below the Fermi le and at absolute temperature *T*=0

$$E - E_F \ll 1$$

$$f(E) = \frac{1}{1 + exp\left[\frac{-(E - E_F)}{\mathrm{k_B}\, T}\right]} = \frac{1}{1 + exp(-\infty)} = \frac{1}{1 + \frac{1}{exp(\infty)}} = \frac{1}{1 + \frac{1}{\infty}} = 1$$

Case 2: the probability of finding electron above the Fermi level and at absolute temperature *T*=0

$$f(E) = \frac{1}{1 + exp\left[\frac{(E - E_F)}{\mathrm{k_B}\, T}\right]} = \frac{1}{1 + exp(\infty)} = \frac{1}{1 + \infty} = 0$$

Case 3: the probability of finding an electron on the Fermi level

$$E - E_F = 0$$

$$f(E) = \frac{1}{1 + exp\left[\frac{0}{\mathrm{k_B}\, T}\right]} = \frac{1}{1 + exp(0)} = \frac{1}{1 + 1} = \frac{1}{2}$$

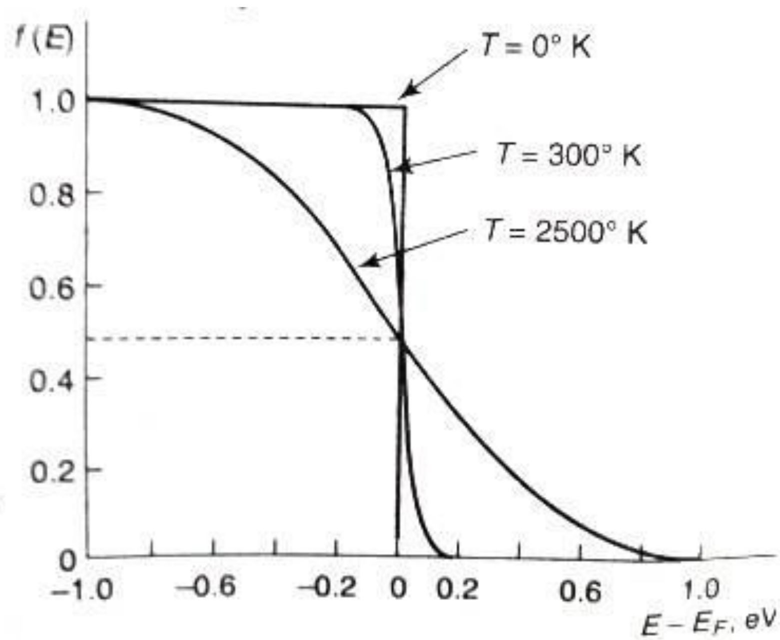


Figure 22.

Note: Fermi-Dirac distribution only gives the probability of occupancy of the state at a given energy level but doesn't provide any information about the number of states available at that energy level.

Example: In a solid consider the energy level lying 0.11 eV below the Fermi level. Find the probability of this level not being occupied by the electron at room temperature.

Solution:

$(E - E_F) = 0.11\ eV$, and $\mathrm{k_B}T$ = 0.026 eV at room temperature.

$$f(E) = \frac{1}{1 + exp\left[\frac{-(E - E_F)}{\mathrm{k_B}\ T}\right]} = \frac{1}{1 + exp\left(\frac{-0.11}{0.026}\right)} = \frac{1}{1.0145} = 0.9857$$

The probability of not finding the electron is, p= $1 - f(E)$ = 0.0146